$c_f = 2\tau_w/\rho V^2$ is the frictional drag coefficient;
c_{f0} is the frictional drag coefficient of Newtonian fluid in stabilized flow region.

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THERMOCONVECTION WAVES IN ASYMMETRICAL FLUIDS

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The propagation of thermoconvection waves in asymmetrical fluids is investigated. The results lead to a number of conclusions about the influence of microinertia and couple stresses on the wave propagation velocity and damping.

Lykov and Berkovskii [1, 2] have investigated the propagation of thermoconvection waves in viscous and viscoelastic fluids. Listrov and Shurinov [3] have studied the propagation of small shear disturbances in certain asymmetrical media. Here we consider the propagation of thermoconvection waves in asymmetrical fluids, using the equations of motion with regard for compressibility in the form [4, 5]

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0, \tag{1}$$

$$\rho \frac{d\mathbf{v}}{dt} = -\operatorname{grad} p + k \operatorname{rot} \boldsymbol{\omega} - (\mu + k) \operatorname{rot} \operatorname{rot} \mathbf{v} + (\lambda + 2\mu + k) \operatorname{grad} \operatorname{div} \mathbf{v} + \rho \mathbf{g}, \tag{2}$$

$$\rho J \frac{d\boldsymbol{\omega}}{dt} = -2k\boldsymbol{\omega} + k \operatorname{rot} \mathbf{v} - \gamma \operatorname{rot} \operatorname{rot} \boldsymbol{\omega} + (\alpha + \beta + \gamma) \operatorname{grad} \operatorname{div} \boldsymbol{\omega}. \tag{3}$$

The tensile stresses t_{ij} and couple stresses m_{ij} are determined from the rheological equations

$$t_{ij} = (-p + \lambda \operatorname{div} \mathbf{v}) \delta_{ij} + \left(\mu + \frac{k}{2}\right)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + k\varepsilon_{ijm}\left(\frac{1}{2}\varepsilon_{mrl}\frac{\partial v_l}{\partial x_r} - \omega_m\right), \tag{4}$$

$$m_{ij} = \alpha (\operatorname{div} \boldsymbol{\omega}) \delta_{ij} + \beta \frac{\partial \omega_i}{\partial x_j} + \gamma \frac{\partial \omega_j}{\partial x_i}. \tag{5}$$

We write the heat-transfer equation in the form [6]

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_k \frac{\partial T}{\partial x_k}\right) = \theta \Delta T + R, \tag{6}$$

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where R is the dissipation function

$$R = \frac{\partial v_i}{\partial x_j} t_{ij} + \frac{\partial \omega_i}{\partial x_j} m_{ij} + \varepsilon_{ijk} t_{jk} \omega_i. \tag{7}$$

Equations (1)-(3), (6) form a closed system if we complement it with the equation of state

$$f(p, \rho, T) = 0. \tag{8}$$

Suppose now that a constant temperature gradient $x \geq 0$, antiparallel to the gravitational field, exists in a fluid occupying the halfspace $x \geq 0$ and having a plane free boundary $x = 0$. Let us assume that the absolute value of the gradient is less than the critical value at which convection sets in. We wish to analyze the propagation of small temperature, velocity, pressure, and density perturbations in such a system.

We seek a solution of the system (1)-(3), (6), (8) in the form

$$\begin{gathered} T = T_0 + T'(x, t), \quad v_x = 0 + v_x'(x, t), \\ v_y = 0 + v_y'(x, t), \quad \omega_z = 0 + \omega_z'(x, t), \\ p = p_0 + p'(x, t), \quad \rho = \rho_0 + \rho'(x, t). \end{gathered} \tag{9}$$

Here T_0, p_0, ρ_0 are the equilibrium temperature, pressure, and density distributions, and T', p', ρ', v_x', v_y', ω_z' are small perturbations. Substituting expressions (9) into (1)-(3), (6), (8) and neglecting higher than first-order small quantities, we obtain the linear system

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial v_x'}{\partial x} = 0, \tag{10}$$

$$\rho_0 c_p \left(\frac{\partial T'}{\partial t} + q v_y' \right) = \theta \frac{\partial^2 T'}{\partial x^2} - p_0 \frac{\partial v_x'}{\partial x}, \tag{11}$$

$$\rho' = \left(\frac{\partial \rho}{\partial T} \right)_{T=T_0} T', \tag{12}$$

$$\rho_0 \frac{\partial v_x'}{\partial t} = -\frac{\partial p'}{\partial x} + (\lambda + k) \frac{\partial^2 v_x'}{\partial x^2}, \tag{13}$$

$$\rho_0 \frac{\partial v_y'}{\partial t} = (\mu + k) \frac{\partial^2 v_y'}{\partial x^2} - k \frac{\partial \omega_z'}{\partial x} - g \left(\frac{\partial \rho}{\partial T} \right)_{T=T_0} T', \tag{14}$$

$$\rho_0 J \frac{\partial \omega_z'}{\partial t} = -2k\omega_z' + k \frac{\partial v_y'}{\partial x} + \gamma \frac{\partial^2 \omega_z'}{\partial x^2}. \tag{15}$$

In the derivation of (10)-(15) we have neglected the variation of the density with the pressure and considered ρ_0 to be constant. It follows directly from (10)-(15) that

$$\rho_0 c_p \left(\frac{\partial T'}{\partial t} + q v_y' \right) = \theta \frac{\partial^2 T'}{\partial x^2} + \frac{p_0}{\rho_0} \left(\frac{\partial \rho}{\partial T} \right)_{T=T_0} \frac{\partial T'}{\partial t}, \tag{16}$$

$$\rho_0 \frac{\partial v_y'}{\partial t} = (\mu + k) \frac{\partial^2 v_y'}{\partial x^2} + k \frac{\partial \omega_z'}{\partial x} - g \left(\frac{\partial \rho}{\partial T} \right)_{T=T_0} T', \tag{17}$$

$$\rho_0 J \frac{\partial \omega_z'}{\partial t} = -2k\omega_z' + k \frac{\partial v_y'}{\partial x} + \gamma \frac{\partial^2 \omega_z'}{\partial x^2}. \tag{18}$$

We seek T', v_y', ω_z' in the form of plane waves:

$$\begin{gathered} T' = N \exp[i(\omega t - Kx)], \quad v_y' = V \exp[i(\omega t - Kx)], \\ \omega_z' = \Omega \exp[i(\omega t - Kx)], \end{gathered} \tag{19}$$

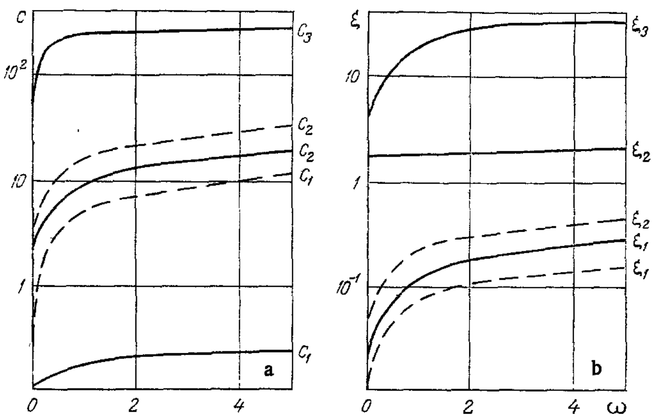

Fig. 1. Wave propagation velocity (a) and damping (b) versus frequency for an asymmetrical fluid (solid curves) and a viscous fluid (dashed curves).

which brings us to a linear homogeneous algebraic system in the perturbation amplitudes:

$$
\begin{gathered}
(ia\omega + bK^2)\,N + qV = 0,\\
dN + (i\omega + fK^2)\,V + iKh\Omega = 0,\\
iKsV + (i\omega + 2s + rK^2)\,\Omega = 0,
\end{gathered}
\tag{20}
$$

where

$$
a = 1 - \frac{p_0}{\rho_0^2 c_p}\left(\frac{\partial\rho}{\partial T}\right)_{T=T_0};\quad b = \theta/\rho_0 c_p;\quad d = \frac{g}{\rho_0}\left(\frac{\partial\rho}{dT}\right)_{T=T_0};
$$

$$
f = (\mu + k)/\rho_0;\quad h = k/\rho_0;\quad s = k/\rho_0 J;\quad r = \gamma/\rho_0 J.
$$

The system (20) is equivalent to the dispersion relation

$$
A(\omega)\,K^6 + B(\omega)\,K^4 + C(\omega)\,K^2 + D(\omega) = 0,
\tag{21}
$$

in which

$$
\begin{gathered}
A(\omega) = bfr,\\
B(\omega) = 2sfb + i\omega\,(br + afr + bf) + hsb,\\
C(\omega) = 2i\omega s\,(af + b) - \omega^2\,(b + ar + af) - qhs - rqd,\\
D(\omega) = -2sqd - qd\omega i - 2sa\omega^2 - ia\omega^3.
\end{gathered}
$$

Equation (21), which describes the relationship between the wave number $K = \omega/c + i\xi$ and the cyclic frequency ω, has been solved numerically on a computer. The following values were assigned to the coefficients in SI units: $J = 5\cdot 10^{-2}$; $\rho_0 = 1$; $g = 9.8$; $p_0 = 1$; $b = 19$; $f = 40$; $h = 15$; $s = 3\cdot 10^2$; $r = 2\cdot 10^2$; $q = 1$; $d = 10^{-2}$; $a = 10^6$, which was chosen so as to comply with the thermodynamic constraints on the transport coefficients of the medium [4, 5]. Curves of the wave propagation velocities $c(\omega)$ and damping factors $\xi(\omega)$ are given in Fig. 1 for symmetrical and viscous ($h = s = r = 0$) fluids, showing that the allowance for microinertia and couple stresses yields a new wave, diminishes the wave propagation velocity, and increases the damping factors.

The conclusions drawn from the given calculations are qualitatively consistent with the existing data on propagation of thermoconvection waves in the case of a viscous heat-conducting fluid [7].

For a viscous fluid we can write (21) in the form

$$
(i\omega a + bK^2)(i\omega + fK^2) - dq = 0,
$$

and as $\omega \to 0$ we obtain $K = K_r + iK_i = \pm\sqrt{\pm\sqrt{dq/bf}}$.

For a thermoconvection wave ($dq > 0$) we have four roots, two of which correspond to damping as $x \to \infty$ ($K_i < 0$): $K_1 = \sqrt[4]{dq/fb}$; $K_2 = -i\sqrt[4]{dq/fb}$. Thus, for $\xi(\omega)$ in Fig. 1, as $\omega \to 0$ the curve $\xi_1 \to 0$, while $\xi_2 \to \sqrt[4]{dq/fb}$. Accordingly, $c_1 \to 0$, and $c_2 \to$ const.

These results can be used to describe thermoconvection phenomena in suspensions, emulsions, and polymer solutions, as well as to determine the viscous, thermal, and inertial properties of fluids.

NOTATION

v	is the velocity of fluid;
ω	is the intrinsic angular velocity of fluid;
ρ	is the density;
p	is the pressure;
t	is the time;
T	is the temperature;
g	is the gravitational acceleration;
c_p	is the isotropic specific heat;
θ	is the thermal conductivity;
J	is the scalar constant with dimensions of moment of inertia per unit mass;
ω	is the cyclic frequency;
K	is the wave number;
$\alpha, \beta, \gamma, \lambda, \mu, k$	are the viscosity coefficients;
δ_{ij}	is the Kronecker delta symbol;
ε_{ijk}	is the Levi-Civita tensor density.

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OSCILLATIONS OF A VISCOELASTIC ROD TAKING THERMOMECHANICAL COUPLING INTO ACCOUNT

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The effect of thermomechanical coupling on the forced longitudinal oscillations of a viscoelastic rod is investigated.

The wide use of viscoelastic materials in many areas of modern technology makes it important to investigate their behavior under different conditions. In this connection, it is of particular interest to study the interaction between the deformation and temperature fields, since viscoelastic materials have the ability to dissipate mechanical energy, and exhibit a considerable temperature dependence of their physicomechanical properties. Consideration of the thermomechanical coupling leads to nonlinearity in the mathematical formulation of the problem, and enables a number of extremely interesting nonlinear effects to be explained. These features of viscoelastic materials manifest themselves most clearly during cyclical deformation. It is shown in [1], using the example of the oscillations of a system with one degree of freedom (a large load on a viscoelastic spring) that over a certain range of variation of the excitation parameter the amplitude–frequency and temperature–frequency dependences are nonunique. These results were confirmed experimentally in [2]. A similar problem was considered in [3] where it was established that for periodic deformations two stable stationary states with different temperatures are possible. In this paper we investigate the effect of thermomechanical coupling on the dynamic behavior of a viscoelastic rod for forced longitudinal oscillations. Subcritical

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